RAN-2203000206020034

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B.Sc. (Sem. - VI) Examination September - 2025

Mathematics (MTH - 604) : (Real Analysis - IV)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) All questions are compulsory. (3) Follow **usual** notations. (4) Figures to the right indicate marks of the question. (5) Total marks **50**.	Student's Signature

Q. 1 Answer any FIVE as directed. [10]

(1) State Heine-Borel property.

(2) Show that a singleton set is closed in any metric space.

(3) Consider [2, 3] in R^1 with absolute value metric, is [2, 3] compact? Justify your answer.

(4) Define Closure of a set E in a metric space M, show that $E \subset \overline{E}$.

(5) Is (0,1) complete in R^1? Justify your answer.

(6) If $A = \{0,2,,4,5,6\} \subset R^1$, where $\delta(x,y) = |x-y|, \forall\, x, y \in A$, then find diameter of A.

(7) State Picard's fixed point Theorem.

(8) If A is Bounded in R^1, then show that A is Totally bounded in R^1.

Q. 2 Attempt any TWO of the following. [10]

(1) If E is a subset of a metric space M, then show that the point $x \in M$ is a limit point of E if and only if every open ball $B[x; r]$ about x contains at least one point of E.

(2) Let $(M_1\delta_1)$ and (M_2,δ_2) be metric spaces. Then show that if f is continuous on M_1 then $f^{-1}(F)$ is closed subset of M_1 whenever F is closed subset of M_2.

(3) Define Dense set in a metric space. Let M be a metric space and let $A \subset B \subset M$. If A is dense in B and B is dense in M, then show that A is dense in M.

Q. 3 Attempt any TWO of the following. [10]

(1) Let M be a metric space. If M is connected then show that every continuous characteristic function on M is constant.

(2) If A_1 and A_2 are connected subsets of a metric space M, and $A_1 \cap A_2 \neq \phi$, then show that $A_1 \cup A_2$ is connected.

(3) If the subset A of a metric space (M,δ) is Totally bounded, then show that A is bounded.

Q. 4 Attempt any TWO of the following. **[10]**

(1) Define Contraction map. Show that a Contraction map in a metric space is continuous.

(2) If (M,δ) is a complete metric space and A is closed subset of M, then show that (A, δ) is complete.

(3) If $T_x = x^2, 0 < x \leq \frac{1}{3}$, then prove that T is a contraction map on $\left[0, \frac{1}{3}\right]$ but T has no fixed point in $\left[0, \frac{1}{3}\right]$

Q. 5 Attempt any TWO of the following. **[10]**

(1) If every sequence of points in a metric space (M, δ) has a subsequence converging to a point in M, then show that the metric space (M, δ) is compact.

(2) If the metric space M is compact then prove that whenever $\mathcal{F}$ is a family of closed subsets of M with finite intersection property, then $\cap_{F \in \mathcal{F}} F \neq \phi$.

(3) If A and B are compact subsets of R^1, then show that A× B is compact in R^2.